第二章　　平面汇交力系和平面力偶系

力系：

平面力系　　平行力系
空间力系　　汇交力系
　　　　　　任意力系

本章主要介绍：

- 平面汇交力系的合成与平衡问题（几何法；解析法）
- 平面力偶系的合成与平衡问题

重点

1、力在坐标轴上的投影，求解平面汇交力系平衡问题的几何法和解析法

2、力偶矩的概念，平面力偶的性质和力偶等效条件

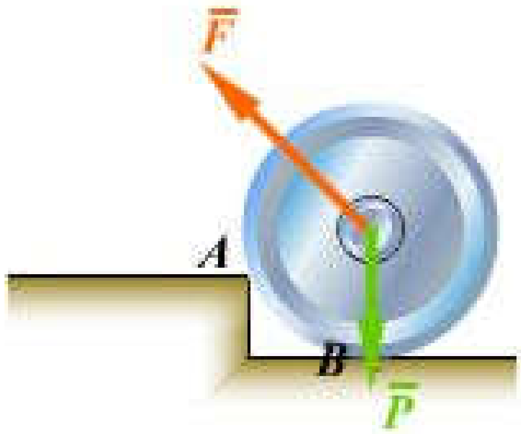
难点

1、平面汇交力系的平衡条件及求解平衡问题的解析法

2、力偶的性质

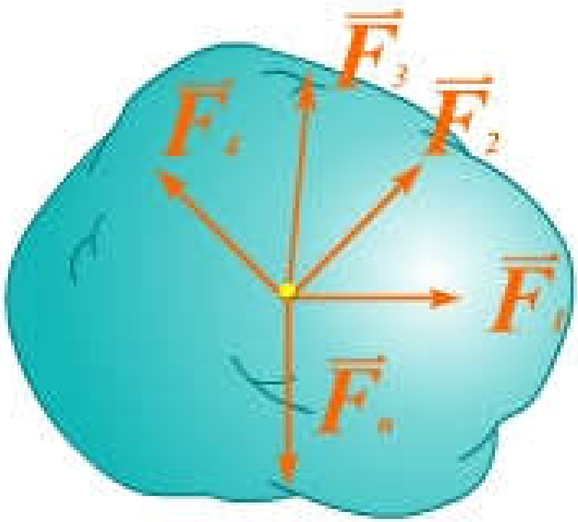
平面汇交力系的概念及工程实例

各力作用线在同一平面内且汇交于一点



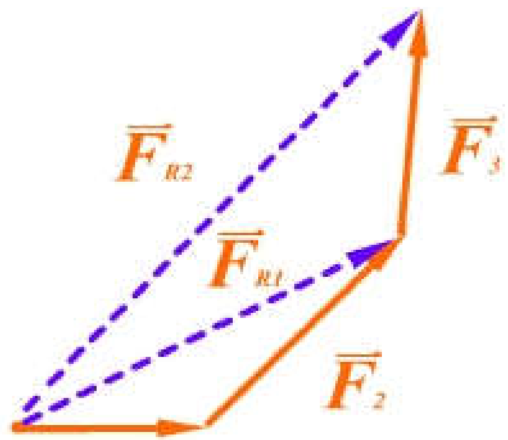
§2-1　平面汇交力系合成与平衡的几何法

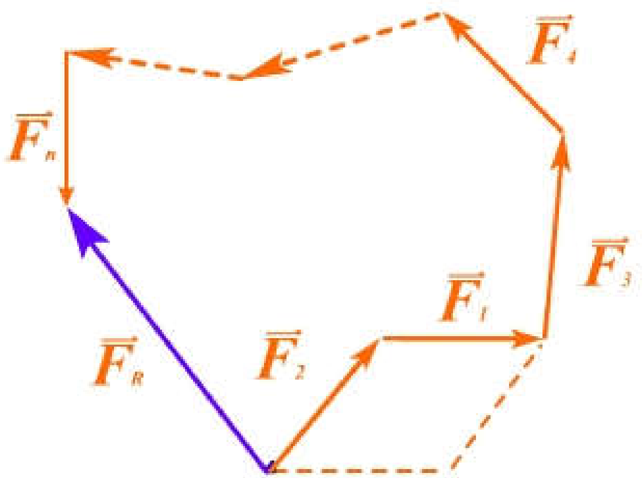
一、多个汇交力的合成 ——— 力多边形规则



$$\vec{F}_{R1} = \vec{F}_1 + \vec{F}_2$$

$$\vec{F}_{R2} = \vec{F}_{R1} + \vec{F}_3 = \sum_{i=1}^{3} \vec{F}_i$$





$$\vec{F}_R = \sum_{i=1}^{n} \vec{F}_i = \boxed{\sum \vec{F}_i \neq \sum F_i}$$

力多边形

力多边形规则

各力首尾相接

二、平面汇交力系平衡的几何条件

平衡条件 $\sum \vec{F}_i = 0$

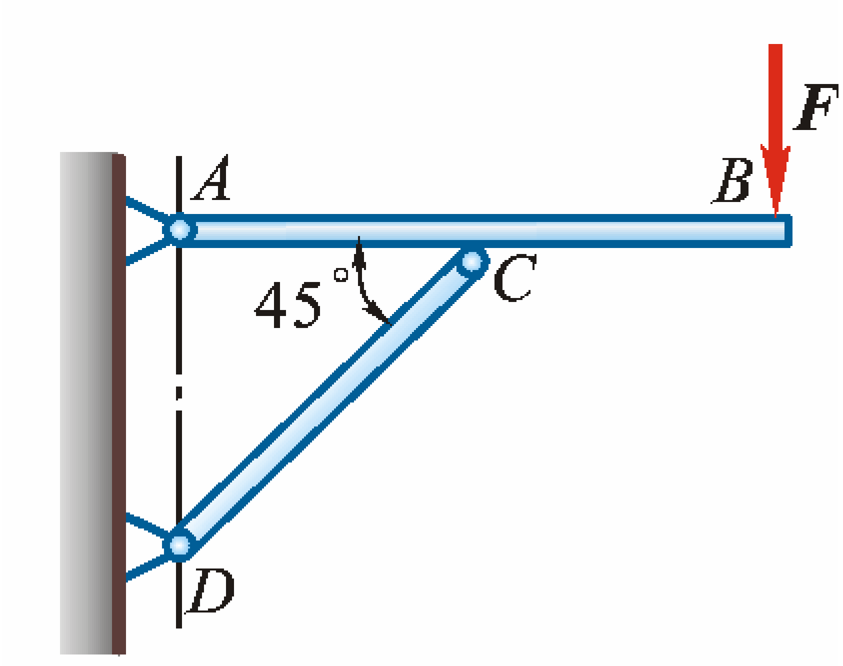
平面汇交力系平衡的必要和充分条件是：

该力系的力多边形自行封闭.

例2-1

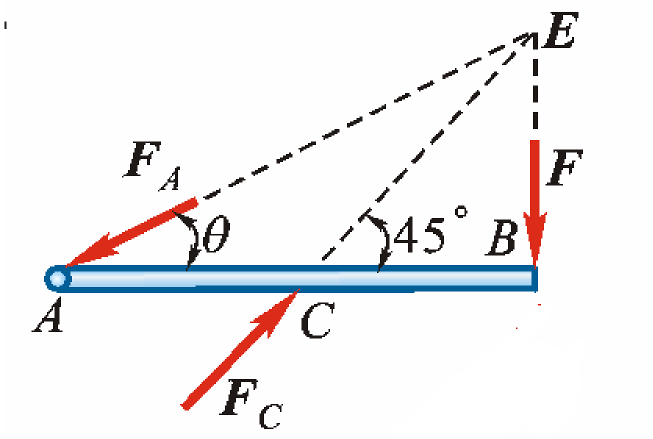
已知： $AC = CB, F = 10\,\text{kN}$ **,各杆自重不计；**

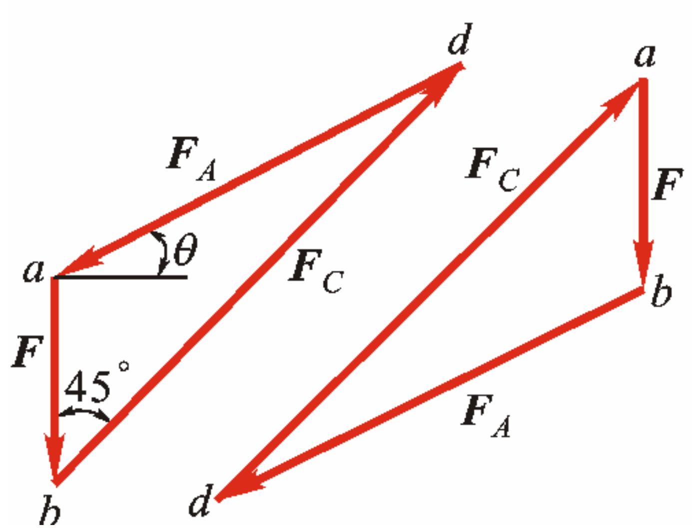
求： CD **杆及铰链** A **的受力.**



解：CD为二力杆，取AB杆，画受力图.

用几何法，画封闭力三角形.





按比例量得 $F_C = 28.3\text{kN}, F_A = 22.4\text{kN}$

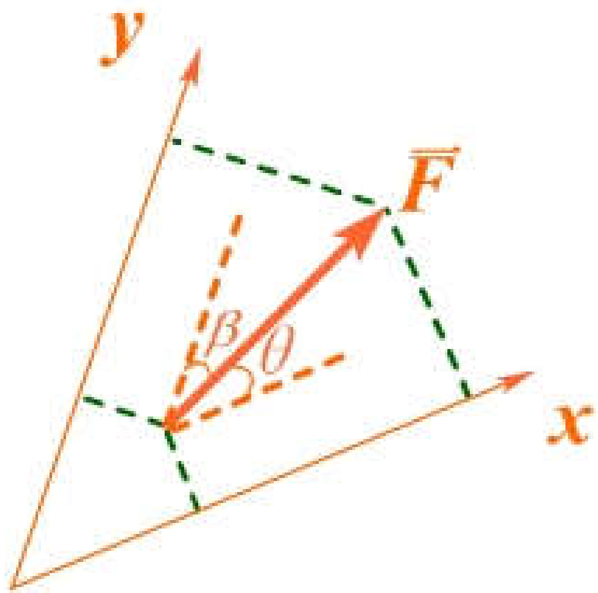
平面汇交力系合成与平衡的几何法，简单、直观，但是当力系中作用力较多时，作图比较麻烦，而且误差可能较大。

下面研究平面汇交力系合成与平衡的解析法。

§2-2　平面汇交力系合成与平衡的解析法

一、力在坐标轴上的投影与力沿轴的分解

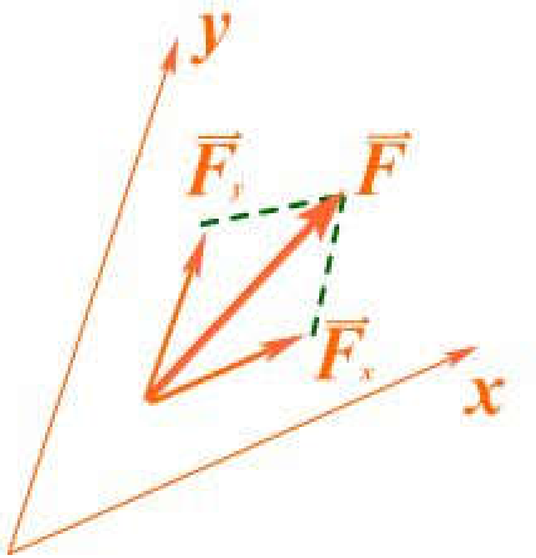
1、力在坐标轴上的投影　(代数量)



$$F_x = F \cdot \cos\theta$$

$$F_y = F \cdot \cos\beta$$

2、力沿轴的分解 （矢量）



$$\vec{F} = \vec{F}_x + \vec{F}_y$$

思考：力的合成、分解是否唯一？

二、平面汇交力系合成的解析法

$$\vec{F}_{\mathrm{R}} = \sum \vec{F}_i$$

由合矢量投影定理，得合力投影定理

$$F_{\mathrm{R}x} = \sum F_{ix} \qquad F_{\mathrm{R}y} = \sum F_{iy}$$

合力的大小为：

$$F_{\mathrm{R}} = \sqrt{F_{\mathrm{R}x}^2 + F_{\mathrm{R}y}^2}$$

方向为：

$$\cos(\vec{F}_{\mathrm{R}}, \vec{i}) = \frac{\sum F_{ix}}{F_{\mathrm{R}}} \qquad \cos(\vec{F}_{\mathrm{R}}, \vec{j}) = \frac{\sum F_{iy}}{F_{\mathrm{R}}}$$

作用点为力的汇交点.

三、平面汇交力系的平衡方程

平衡条件 $\vec{F}_{\mathrm{R}} = 0$

平衡方程

$$\sum F_x = 0$$

$$\sum F_y = 0$$

说明：平面汇交力系沿任意轴投影的代数和为零，则该力系平衡。

例2-2

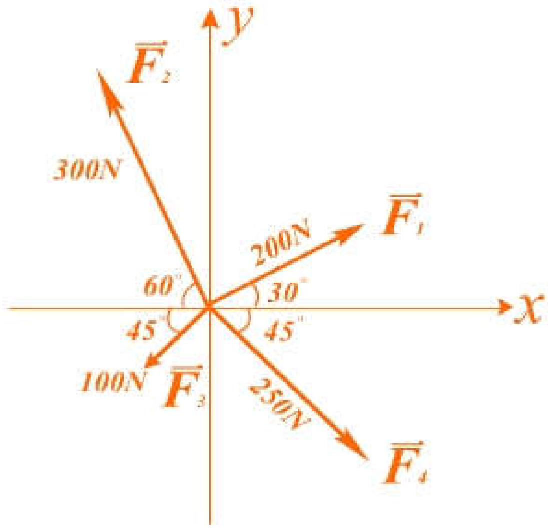
已知：图示平面共点力系；　　求：此力系的合力.

解：用解析法

$$F_{Rx}=\sum F_{ix}=F_1\cos30^\circ-F_2\cos60^\circ-F_3\cos45^\circ+F_4\cos45^\circ=129.3\text{N}$$

$$F_{Ry}=\sum F_{iy}=F_1\sin30^\circ+F_2\sin60^\circ-F_3\sin45^\circ-F_4\sin45^\circ=112.3\text{N}$$

$$F_R=\sqrt{F_{Rx}^2+F_{Ry}^2}=171.3\text{N}$$



$$\cos\theta=\frac{F_{Rx}}{F_R}=0.7548$$

$$\cos\beta=\frac{F_{Ry}}{F_R}=0.6556$$

$$\theta=40.99^\circ,\ \beta=49.01^\circ$$

例2-3

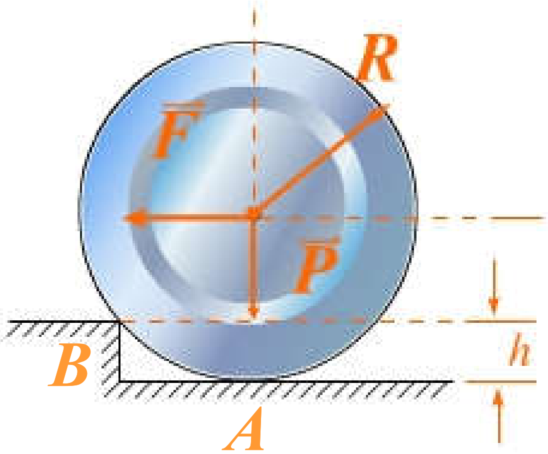
已知： $P = 20\,\text{kN}, R = 0.6\,\text{m}, h = 0.08\,\text{m}$

求：

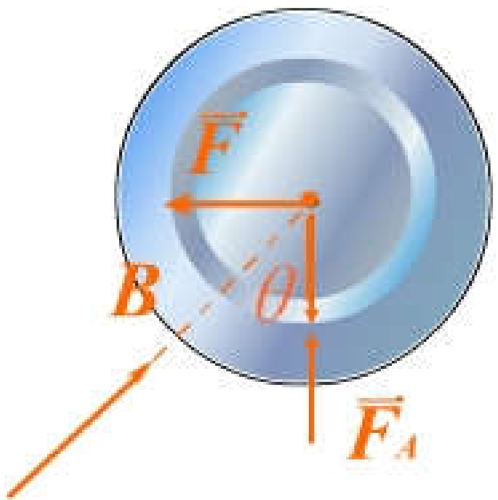
1.水平拉力 $F = 5\,\text{kN}$ **时，碾子对地面及障碍物的压力？**

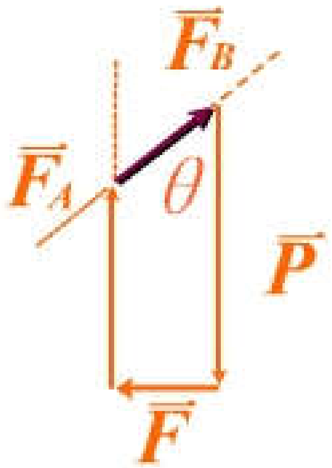
2.欲将碾子拉过障碍物，水平拉力 $\vec{F}$ **至少多大？**

3.力 $\vec{F}$ **沿什么方向拉动碾子最省力，及此时力** $\vec{F}$ **多大？**



解:1.取碾子，画受力图. 用几何法，按比例画封闭力四边形





$$\theta = \arccos\frac{R-h}{R} = 30^\circ$$

$$F_B \sin\theta = F$$

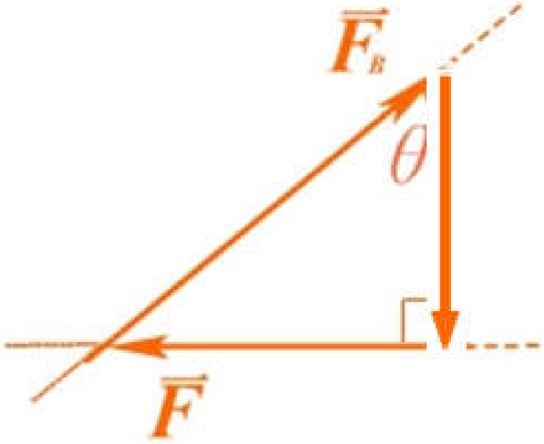
$$F_A + F_B \cos\theta = P$$

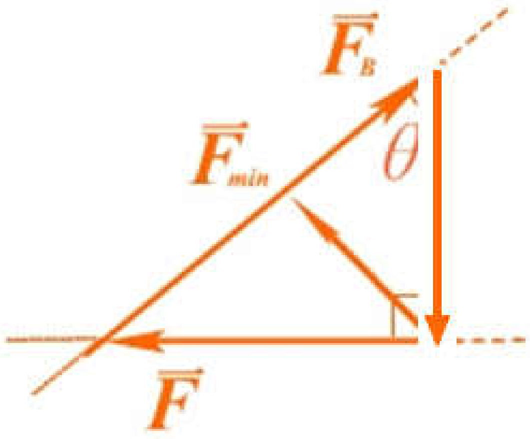
$$F_A = 11.4\text{kN}$$

$$F_B = 10\text{kN}$$

2.碾子拉过障碍物，　应有 $F_A = 0$

用几何法解得　$F = P \cdot \tan\theta = 11.55\text{kN}$



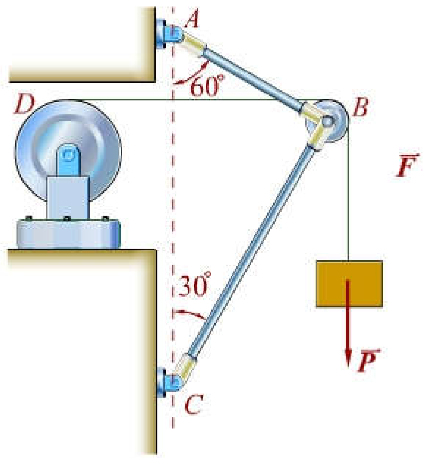


3. 解得 $F_{\min} = P \cdot \sin\theta = 10\ \text{kN}$

例2-4

已知：系统如图，不计杆、轮自重，忽略滑轮大小，P=20kN；

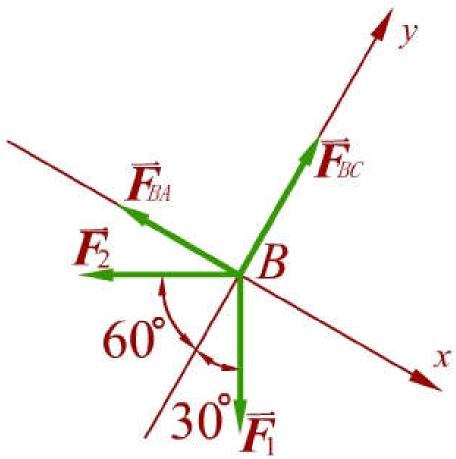
求：系统平衡时，杆*AB*、*BC*受力.



解：*AB*、*BC*杆为二力杆，

取滑轮*B*（或点*B*），画受力图.

用解析法，建图示坐标系



$$\sum F_{ix}=0 \qquad -F_{BA}+F_1\cos60^\circ-F_2\cos30^\circ=0$$

$$F_1=F_2=P$$

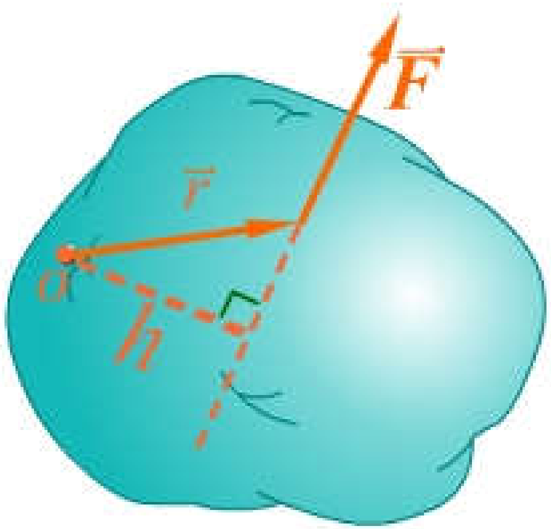
解得： $F_{BA}=-7.321\text{kN}$

$$\sum F_{iy}=0 \qquad F_{BC}-F_1\cos30^\circ-F_2\cos60^\circ=0$$

解得： $F_{BC}=27.32\text{kN}$

§2-3　平面力对点之矩的概念和计算

一、平面力对点之矩（力矩）



力矩作用面，O称为矩心，O到力的作用线的垂直距离h称为力臂

两个要素：

1.大小：力 $\vec{F}$ 与力臂的乘积

2.方向：转动方向

$$M_0(\vec{F}) = \pm F \cdot h$$

$$M_0(\vec{F}) = \pm \left| \vec{r} \times \vec{F} \right|$$

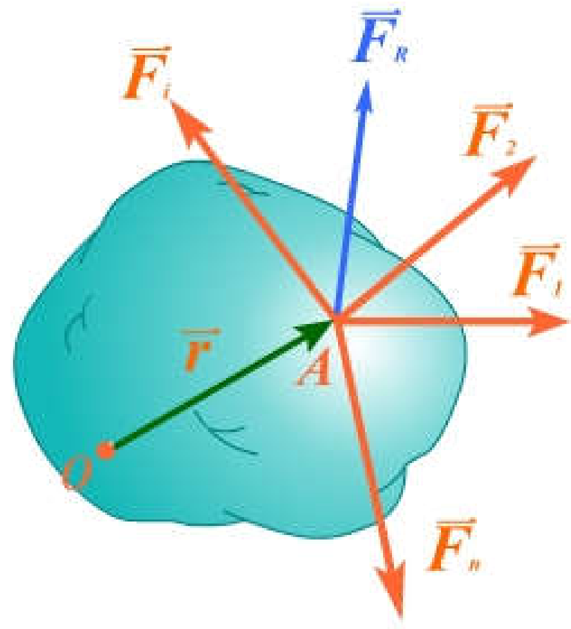
力对点之矩是一个代数量，它的绝对值等于力的大小与力臂的乘积，它的正负：力使物体绕矩心逆时针转向时为正，反之为负。常用单位 $\mathrm{N \cdot m}$ 或 $\mathrm{kN \cdot m}$

二、合力矩定理

$$\vec{F}_R=\sum\vec{F}_i=\vec{F}_1+\vec{F}_2+\cdots+\vec{F}_n$$

$$\vec{F}_R=\vec{F}_1+\vec{F}_2+\cdots+\vec{F}_n$$

$$\vec{r}\times\vec{F}_R=\vec{r}\times\vec{F}_1+\vec{r}\times\vec{F}_2+\cdots+\vec{r}\times\vec{F}_n$$



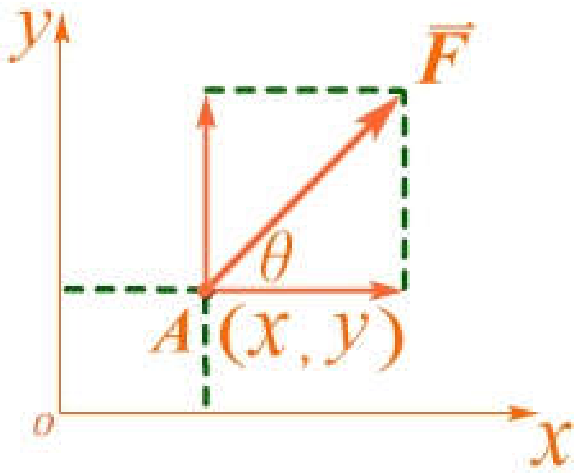
即 $\boxed{\vec{M}_O(\vec{F}_R)=\sum\vec{M}_O(\vec{F}_i)}$

合力矩定理（适用于任何合力存在的力系）

平面汇交力系：$M_0(\vec{F}_R)=\sum M_0(\vec{F}_i)$

三、力矩与合力矩的解析表达式

$$M_O(\vec{F}) = M_O(\vec{F}_y) + M_O(\vec{F}_x)$$
$$= x \cdot F \cdot \sin\theta - y \cdot F \cdot \cos\theta$$
$$= xF_y - yF_x$$



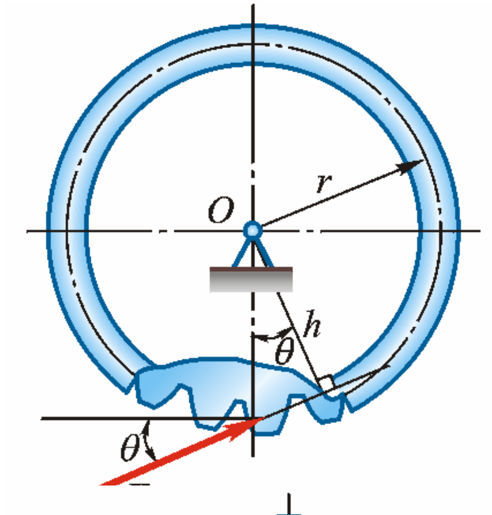
$$M_O\left(\vec{F}_\mathrm{R}\right) = \sum M_O\left(\vec{F}_i\right)$$

$$M_O\left(\vec{F}_\mathrm{R}\right) = \sum\left(x_i \cdot F_{iy} - y_i \cdot F_{ix}\right)$$

例2-5

已知： $F = 1400\text{N}, \quad \theta = 20^{\circ}, \quad r = 60\text{mm}$

求： $M_O(\vec{F})$

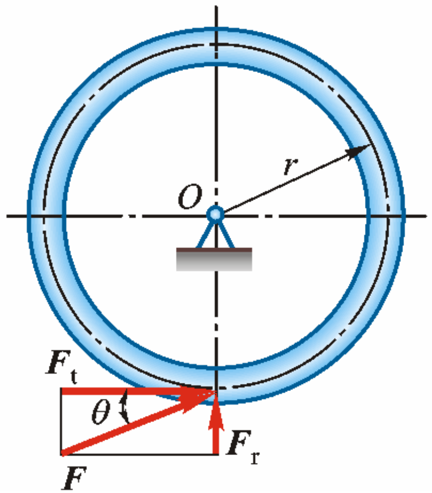


解:直接按定义

$$M_O\left(\vec{F}\right) = F \cdot h = F \cdot r \cdot \cos\theta$$

$$= 78.93\text{N} \cdot \text{m}$$

按合力矩定理



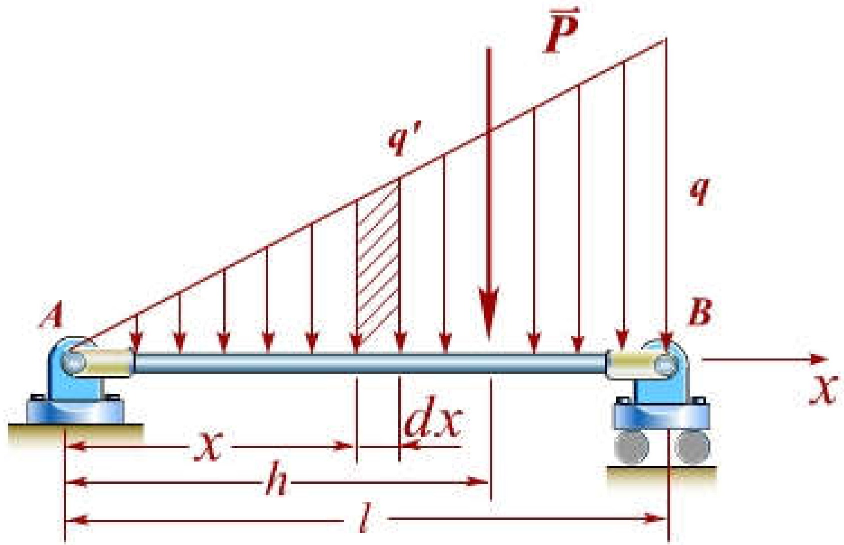
$$M_O\left(\vec{F}\right) = M_O\left(\vec{F}_t\right) + M_O\left(\vec{F}_r\right)$$

$$= F \cdot \cos\theta \cdot r = 78.93\text{N} \cdot \text{m}$$

例2-6 **已知：** $q, l;$

求：合力及合力作用线位置.

解：取微元如图



$$q' = \frac{x}{l} \cdot q$$

$$P = \int_0^l \frac{x}{l} \cdot q \cdot \mathrm{d}x = \frac{1}{2} ql$$

由合力矩定理 $$P \cdot h = \int_0^l q' \cdot \mathrm{d}x \cdot x = \int_0^l \frac{x^2}{l} q \cdot \mathrm{d}x$$

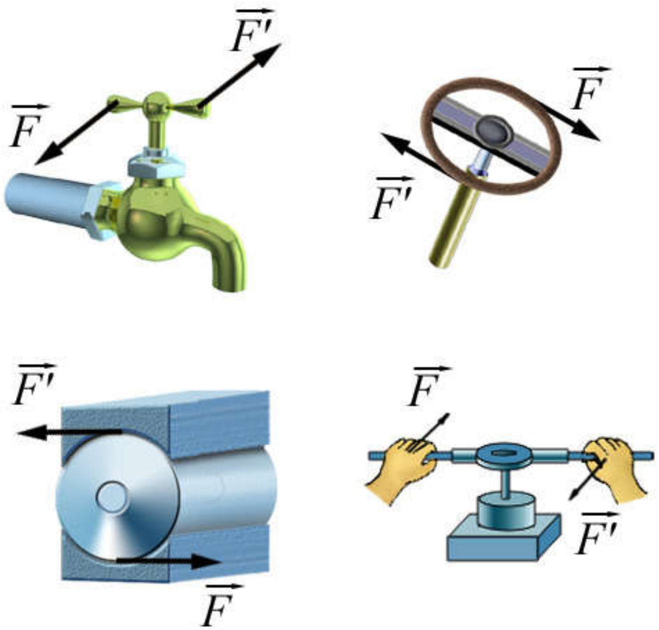
得 $$h = \frac{2}{3} l$$

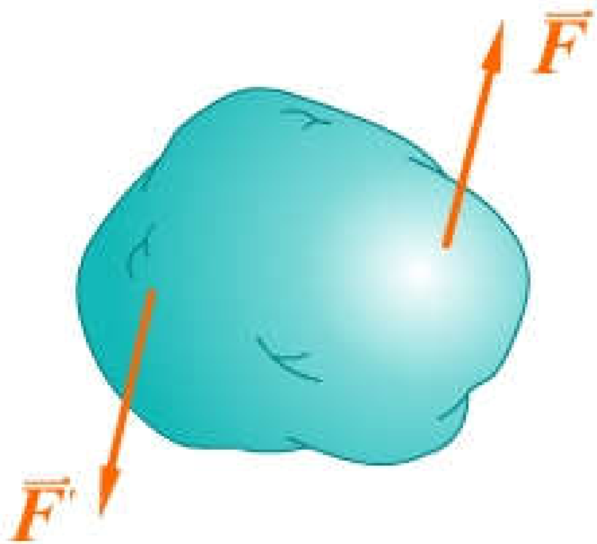
§ 2-4　　平面力偶理论

一、力偶和力偶矩

1.力偶

由两个等值、反向、不共线的平行力组成的力系称为力偶，记作 $\left(\vec{F},\vec{F}'\right)$





2.力偶矩

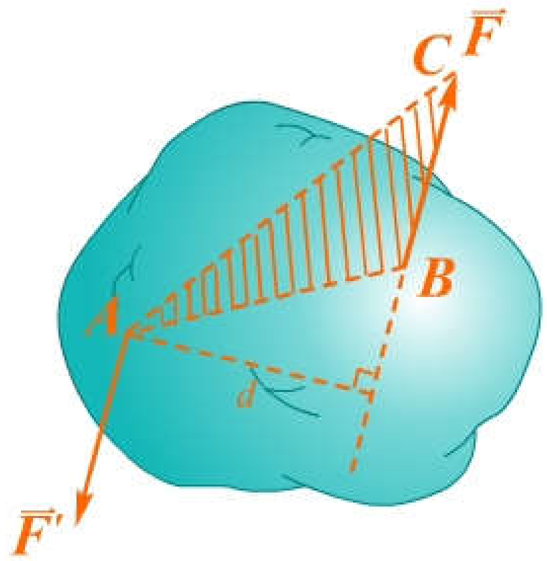
力偶中两力所在平面称为力偶作用面.

力偶两力之间的垂直距离称为力偶臂.

两个要素

a.大小：力与力偶臂乘积

b.方向：转动方向

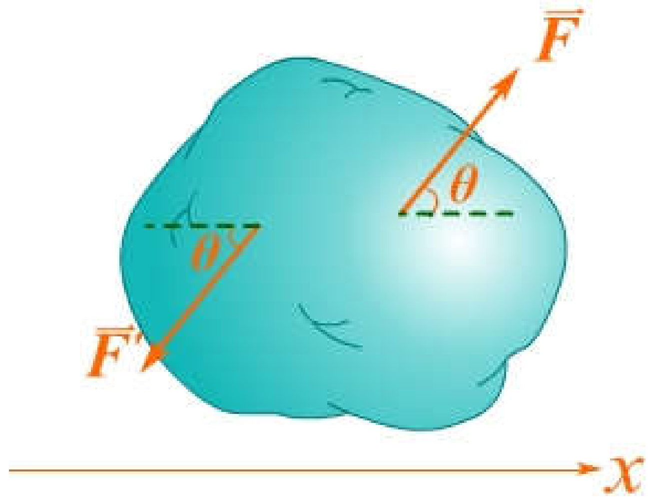


力偶矩 **（代数量）**

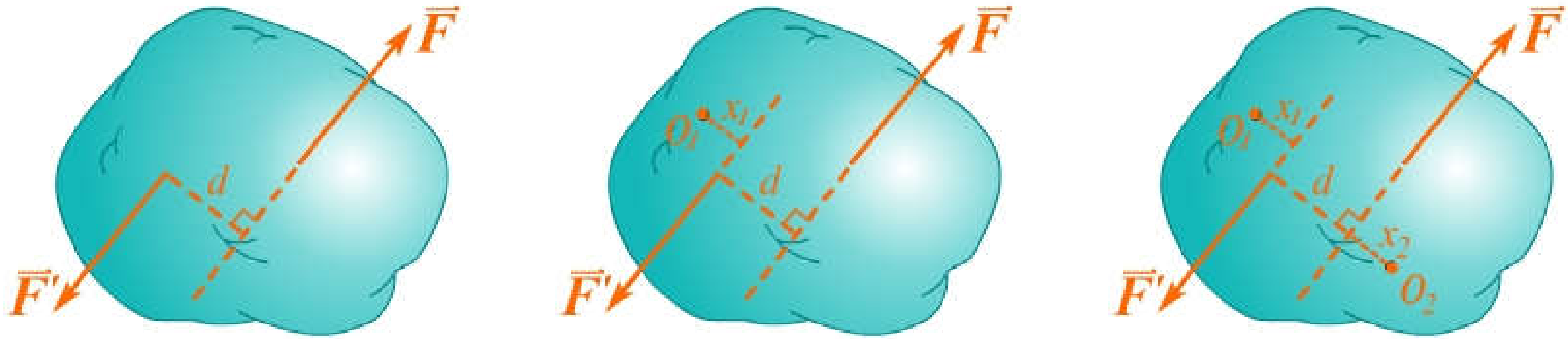
$$M = \pm F \cdot d = \pm 2S_{\Delta ABC}$$

二、力偶与力偶矩的性质

1.力偶在任意坐标轴上的投影等于零.



2.力偶对任意点取矩都等于力偶矩，不因矩心的改变而改变.



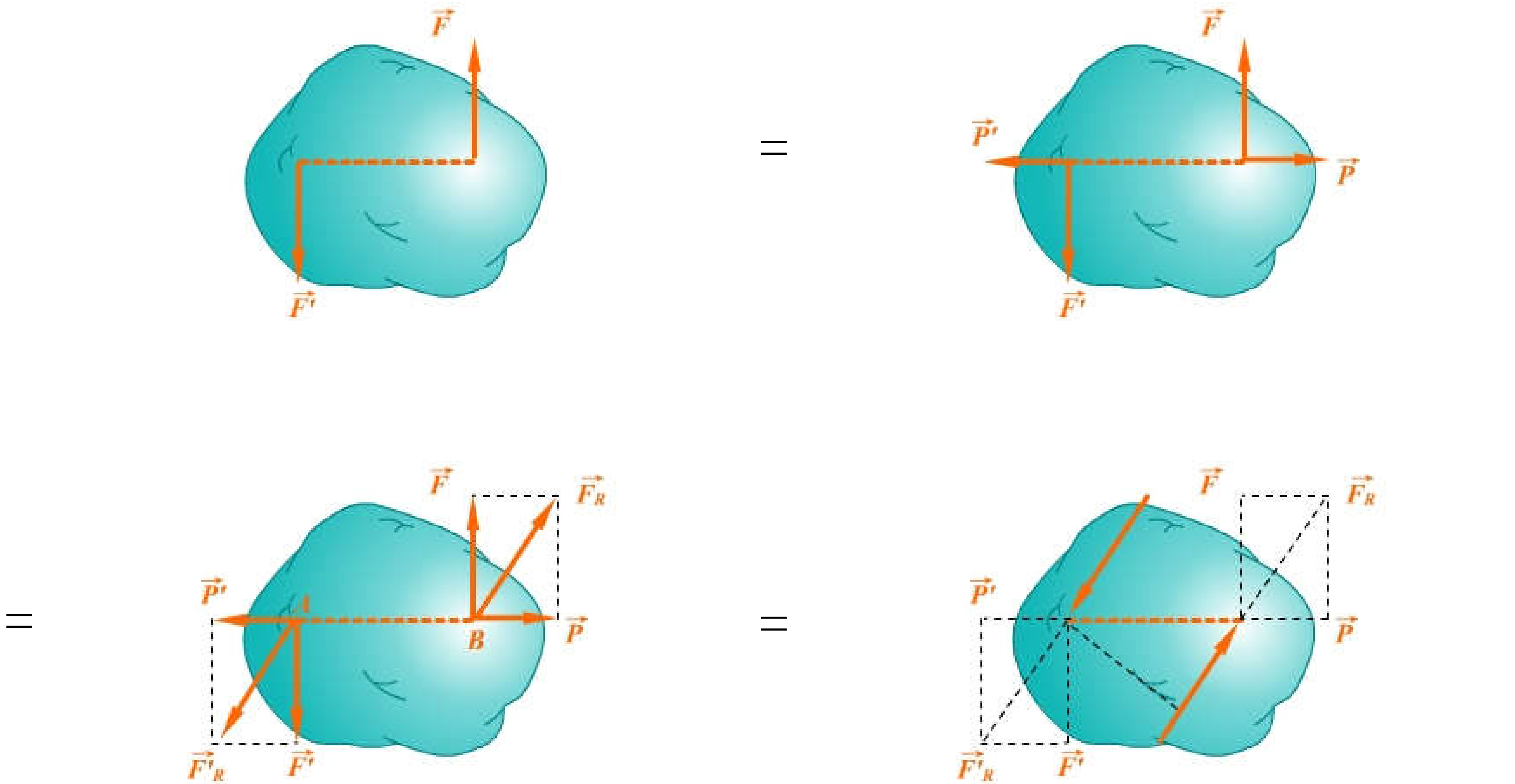
$$M_{O_1}\left(\vec{F},\vec{F}'\right)=M_{O_1}\left(\vec{F}\right)+M_{O_1}\left(\vec{F}'\right)$$
$$=F\cdot\left(d+x_1\right)-F\cdot x_1=Fd$$

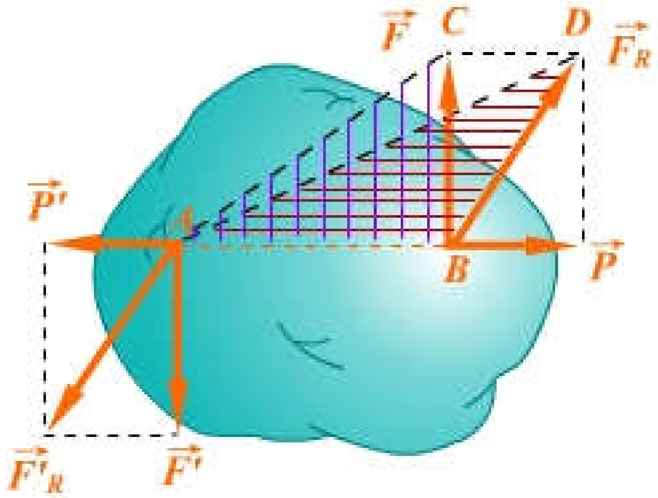
$$M_{O_2}\left(\vec{F},\vec{F}'\right)=F'\cdot\left(d+x_2\right)-F\cdot x_2$$
$$=F'd=Fd$$

力偶矩的符号

M

3.只要保持力偶矩不变，力偶可在其作用面内任意移转，且可以同时改变力偶中力的大小与力偶臂的长短，对刚体的作用效果不变.



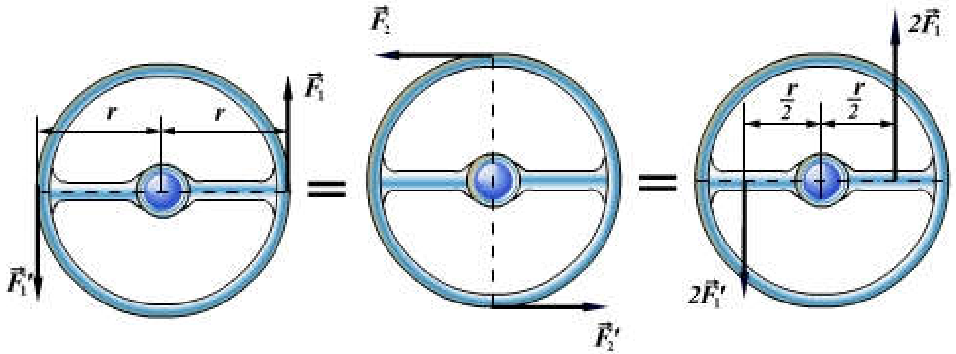


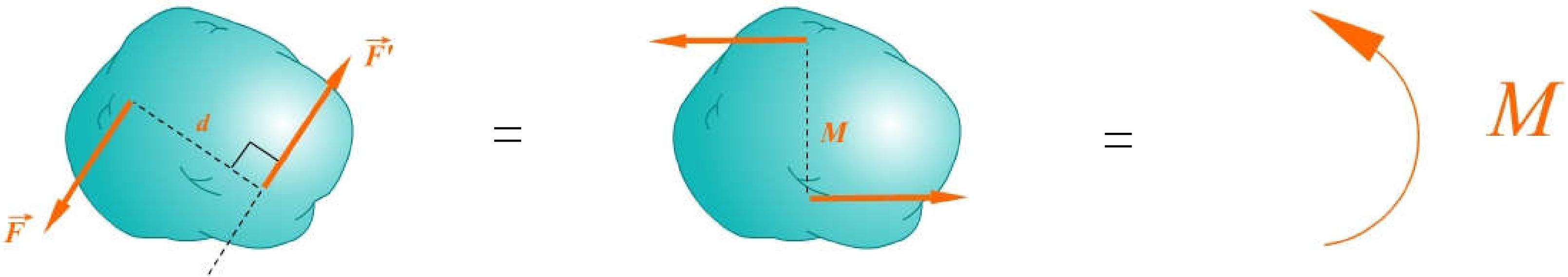
$$S_{\Delta ABC} \ ? \ S_{\Delta ABD}$$

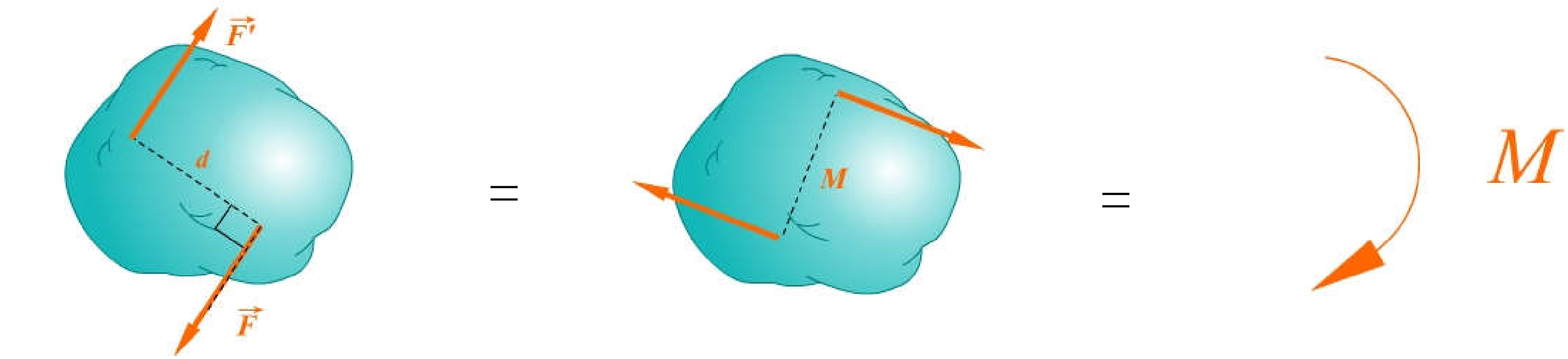
$$S_{\Delta ABC} = S_{\Delta ABD}$$

$$M\left(\vec{F}_{\mathrm{R}}, \vec{F}'_{\mathrm{R}}\right) = F_{\mathrm{R}} d_1 = 2S_{\Delta ABD}$$

$$M\left(\vec{F}, \vec{F}'\right) = Fd = 2S_{\Delta ABC}$$


$\vec{F}_1$
r
r
$\vec{F}_1'$
$\vec{F}_2$
$\vec{F}_2'$
$2\vec{F}_1$
$\frac{r}{2}$
$\frac{r}{2}$
$2\vec{F}_1'$


$\vec{F}'$
d
$\vec{F}$
=
M
=
M


$\vec{F}'$
d
$\vec{F}$
=
M
=
M

4.力偶没有合力，力偶只能由力偶来平衡.

小结

1.力和力偶

从作用效果看：力偶使刚体产生转动；力可使刚体产生移动+转动.

从量度看：力偶在任意坐标轴上的投影为零；力偶只能用力偶平衡，力则可用力或力偶来平衡.

2.力矩和力偶矩

同为度量转动效应的物理量.力偶矩与矩心位置无关.

三、平面力偶系的合成和平衡条件

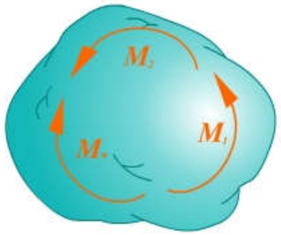
已知： $M_1, M_2, \cdots M_n$;

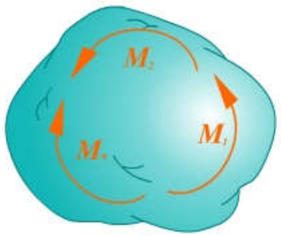
任选一段距离 d

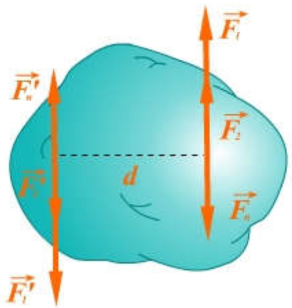
$$\frac{M_1}{d} = F_1 \qquad M_1 = F_1 d$$

$$\frac{M_2}{d} = F_2 \qquad M_2 = F_2 d$$

$$\left|\frac{M_n}{d}\right| = F_n \qquad M_n = -F_n d$$







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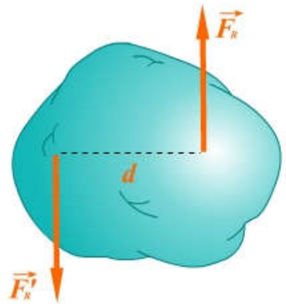
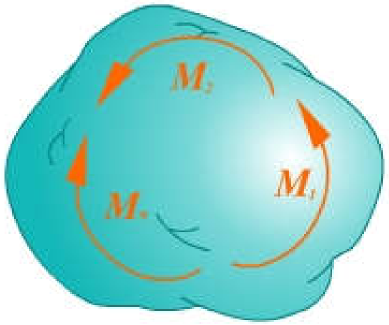


$$F_{\mathrm{R}} = F_1 + F_2 + \cdots - F_n$$

$$F'_{\mathrm{R}} = F'_1 + F'_2 + \cdots - F'_n$$



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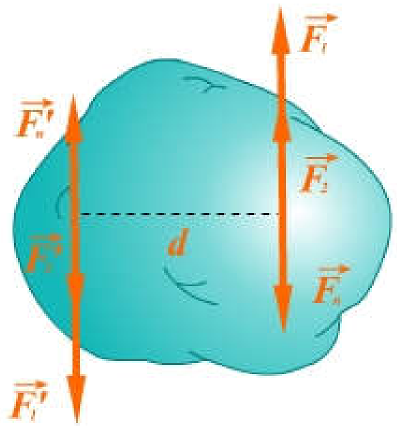

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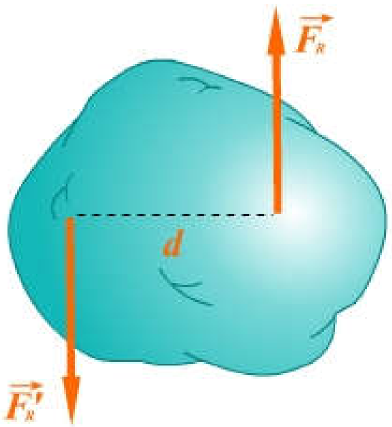

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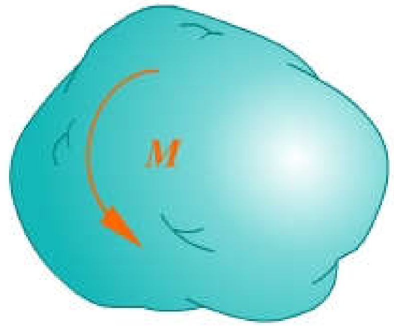


$$M = F_R d = F_1 d + F_2 d + \cdots - F_n d = M_1 + M_2 + \cdots M_n$$

$$M = \sum_{i=1}^{n} M_i = \sum M_i$$

平面力偶系平衡的充要条件 $M = 0$**，有如下平衡方程**

$$\sum M_i = 0$$

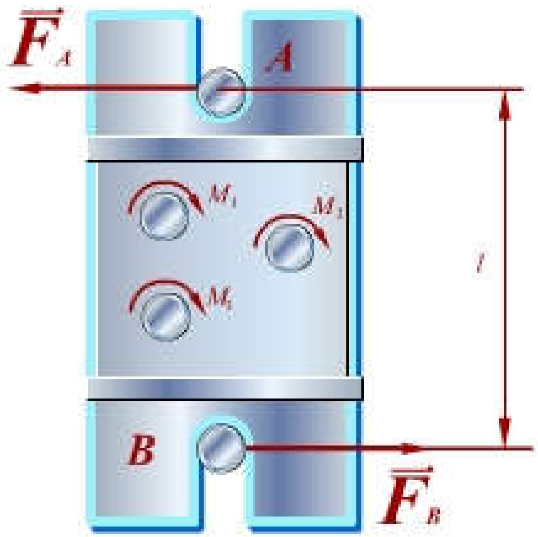
平面力偶系平衡的必要和充分条件是：

所有各力偶矩的代数和等于零.

例2-7

已知： $M_1 = M_2 = 10\ \mathbf{N \cdot m}, M_3 = 20\ \mathbf{N \cdot m}, l = 200\ \mathbf{mm}$；

求： 光滑螺柱 A所受水平力.



解：由力偶只能由力偶平衡的性质，其受力图为

$$\sum M = 0$$

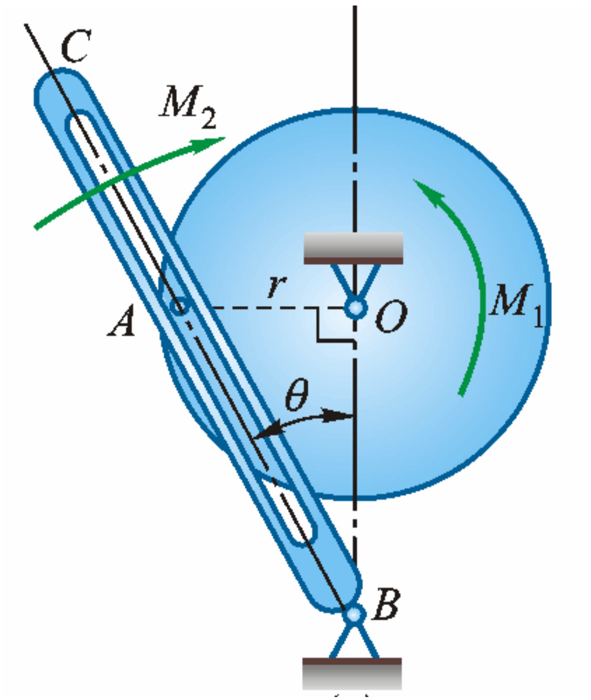
$$F_A l - M_1 - M_2 - M_3 = 0$$

解得 $$F_A = F_B = \frac{M_1 + M_2 + M_3}{l} = 200\ \mathrm{N}$$

例2-8

已知 $M_1 = 2\text{kN}\cdot\text{m}, OA = r = 0.5\text{m}, \theta = 30^\circ;$

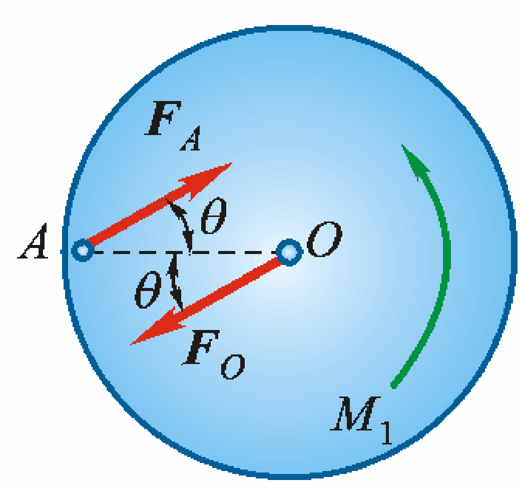
求：平衡时的 M_2 及铰链 O,B 处的约束力.



解：取轮,由力偶只能由力偶平衡的性质,画受力图.

$$\sum M = 0 \qquad M_1 - F_A \cdot r \sin\theta = 0$$

解得 $\quad F_O = F_A = 8\text{kN}$



取杆 BC **画受力图.**

$$\sum M = 0 \qquad F_A' \cdot \frac{r}{\sin\theta} - M_2 = 0$$

解得 $\quad M_2 = 8\text{kN} \cdot \text{m}$

$$F_B = F_A = 8\text{kN}$$