

# 牛顿环实验的一种新的数据处理方法

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**摘要** 本文给出了用牛顿环实验测平凸透镜的曲率半径的一种数据处理方法, 并对实验测量的曲率半径作了不确定度评定。

**关键词** 不确定度; 曲率半径

## A NEW METHOD OF DATA TREATMENT IN NEWTON'S RINGS EXPERIMENT

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**Abstract** A data processing method for measuring the curvature radius of plane-convex lens with Newton's rings experiment is introduced. The uncertainty evaluation of the curvature radius measurement is given.

**Key Words** uncertainty; curvature radius

### 1 实验数据处理的方法

#### 1.1 曲率半径 $R$ 的计算公式

考虑到实验中移测显微镜的十字叉丝不易对准牛顿环圆心, 不妨假设圆心对应的标尺读数为  $x_0$ , 从第  $m$  环开始测量, 转动测微鼓轮, 使镜筒沿着同一方向移动, 每隔两环测量一次, 共测 12 次, 每次测量十字叉丝所对应

环的标尺读数分别为  $x_m, x_{m+2}, x_{m+4}, \dots, x_{m+22}$ , 每环所对应的半径分别为

$$|x_m - x_0| = \sqrt{mR\lambda} \quad (1)$$

$$|x_{m+2} - x_0| = \sqrt{(m+2)R\lambda} \quad (2)$$

$$|x_{m+4} - x_0| = \sqrt{(m+4)R\lambda} \quad (3)$$

$$\vdots \quad \vdots \quad \vdots$$

$$|x_{m+22} - x_0| = \sqrt{(m+22)R\lambda} \quad (12)$$

各式两边同时平方得

$$(x_m - x_0)^2 = mR\lambda \quad (13)$$

$$(x_{m+2} - x_0)^2 = (m+2)R\lambda \quad (14)$$

$$(x_{m+4} - x_0)^2 = (m+4)R\lambda \quad (15)$$

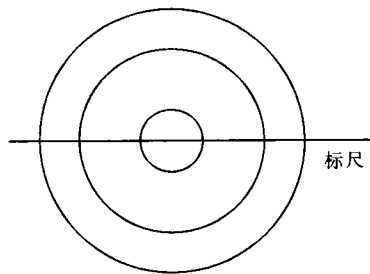
$$\vdots \quad \vdots \quad \vdots$$

$$(x_{m+22} - x_0)^2 = (m+22)R\lambda \quad (24)$$

对于式(13)至式(24)隔六项一次逐差有

$$x_{m+12}^2 - x_m^2 - 2x_0(x_{m+12} - x_m) = 12R\lambda \quad (25)$$

$$x_{m+14}^2 - x_{m+2}^2 - 2x_0(x_{m+14} - x_{m+2}) = 12R\lambda \quad (26)$$



$$x_{m+16}^2 - x_{m+4}^2 - 2x_0(x_{m+16} - x_{m+4}) = 12R\lambda \quad (27)$$

$$\vdots \quad \vdots \quad \vdots$$

$$x_{m+22}^2 - x_{m+10}^2 - 2x_0(x_{m+22} - x_{m+10}) = 12R\lambda \quad (30)$$

整理得

$$x_{m+12} + x_m - 2x_0 = \frac{12R\lambda}{x_{m+12} - x_m} \quad (31)$$

$$x_{m+14} + x_{m+2} - 2x_0 = \frac{12R\lambda}{x_{m+14} - x_{m+2}} \quad (32)$$

$$x_{m+16} + x_{m+4} - 2x_0 = \frac{12R\lambda}{x_{m+16} - x_{m+4}} \quad (33)$$

$$\vdots \quad \vdots \quad \vdots$$

$$x_{m+22} + x_{m+10} - 2x_0 = \frac{12R\lambda}{x_{m+22} - x_{m+10}} \quad (36)$$

再对式(31)至式(36)隔三项逐差得

$$\begin{aligned} & [x_{m+18} + x_{m+6} - (x_{m+12} + x_m)] \\ &= \frac{12R\lambda}{x_{m+18} - x_{m+6}} - \frac{12R\lambda}{x_{m+12} - x_m} \\ &= 12R\lambda \frac{(x_{m+12} - x_m) - (x_{m+18} - x_{m+6})}{(x_{m+18} - x_{m+6})(x_{m+12} - x_m)} \end{aligned}$$

所以曲率半径为

$$R_1 = \frac{1}{12\lambda} \cdot \frac{(x_{m+18} + x_{m+6} - x_{m+12} - x_m)(x_{m+18} - x_{m+6})(x_{m+12} - x_m)}{x_{m+12} + x_{m+6} - x_m - x_{m+18}} \quad (37)$$

同理有

$$R_2 = \frac{1}{12\lambda} \cdot \frac{(x_{m+20} + x_{m+8} - x_{m+14} - x_{m+2})(x_{m+20} - x_{m+8})(x_{m+14} - x_{m+2})}{x_{m+14} + x_{m+8} - x_{m+2} - x_{m+20}} \quad (38)$$

$$R_3 = \frac{1}{12\lambda} \cdot \frac{(x_{m+22} + x_{m+10} - x_{m+16} - x_{m+4})(x_{m+22} - x_{m+10})(x_{m+16} - x_{m+4})}{x_{m+16} + x_{m+10} - x_{m+4} - x_{m+22}} \quad (39)$$

$$\bar{R} = \frac{1}{3} \sum_{i=1}^3 R_i \quad (40)$$

## 1.2 不确定度表达式

### (1) A类不确定度

$$\begin{aligned} U_A &= \frac{t}{\sqrt{n}} \sqrt{\frac{\sum_{i=1}^3 (R_i - \bar{R})^2}{n-1}} \\ &= 2.48 \sqrt{\frac{\sum_{i=1}^3 (R_i - \bar{R})^2}{3-1}} \quad (41) \\ &= 2.48 \sqrt{\frac{\sum_{i=1}^3 (R_i - \bar{R})^2}{2}} \end{aligned}$$

### (2) B类不确定度

$$\text{因为 } \bar{R} = \frac{1}{3}(R_1 + R_2 + R_3)$$

所以

$$U_B(\bar{R}) = \frac{1}{3} \sqrt{U_B^2(R_1) + U_B^2(R_2) + U_B^2(R_3)}$$

假设  $U_B(R_1) = U_B(R_2) = U_B(R_3)$

则

$$U_B(\bar{R}) = \frac{\sqrt{3}}{3} U_B(R_1) \quad (42)$$

由式(37)得

$$\begin{aligned} \ln R_1 &= \ln(x_{m+18} + x_{m+6} - x_{m+12} - x_m) \\ &+ \ln(x_{m+18} - x_{m+6}) + \ln(x_{m+12} - x_m) \\ &- \ln(x_{m+12} + x_{m+6} - x_m - x_{m+18}) - \ln 12\lambda \\ U_{Br}(R_1) &= \frac{U_B(R_1)}{R_1} \\ &= \left[ \left( \frac{1}{x_{m+12} + x_{m+6} - x_m - x_{m+18}} \right. \right. \\ &- \frac{1}{x_{m+12} - x_m} \\ &- \frac{1}{x_{m+18} + x_{m+6} - x_{m+12} - x_m} \left. \right)^2 U_B^2(x_m) \\ &+ \left( \frac{1}{x_{m+18} + x_{m+6} - x_{m+12} - x_m} \right. \end{aligned}$$

$$\begin{aligned}
& - \frac{1}{x_{m+18} - x_{m+6}} \\
& - \frac{1}{x_{m+12} + x_{m+6} - x_m - x_{m+18}} \Big)^2 U_B^2(x_{m+6}) \\
& + \left( \frac{1}{x_{m+12} - x_m} \right. \\
& - \frac{1}{x_{m+12} + x_{m+6} - x_m - x_{m+18}} \\
& - \frac{1}{x_{m+18} + x_{m+6} - x_{m+12} - x_m} \Big)^2 U_B^2(x_{m+12}) \\
& + \left( \frac{1}{x_{m+18} + x_{m+6} - x_{m+12} - x_m} \right. \\
& + \frac{1}{x_{m+18} - x_{m+6}} \\
& + \left. \frac{1}{x_{m+12} + x_{m+6} - x_m - x_{m+18}} \right)^2 U_B^2(x_{m+18}) \Big]^{1/2} \quad (43)
\end{aligned}$$

表 1

环次	$m$	$m+2$	$m+4$	$m+6$	$m+8$	$m+10$	$m+12$	$m+14$	$m+16$	$m+18$	$m+20$	$m+22$
标尺读数 $x(\text{mm})$	24.055	23.701	23.389	23.122	22.853	22.592	22.384	22.114	21.888	21.670	21.474	21.266

## 2.2 数据处理

### (1) 曲率半径 $\bar{R}$

由式(37)至式(40)得

$$\begin{aligned}
\bar{R} &= \frac{1}{3} \sum_{i=1}^3 R_i \\
&= \frac{1}{3} (2.5803 + 2.2139 + 2.2822) = 2.3588\text{m}
\end{aligned}$$

### (2) $\bar{R}$ 的 A 类不确定度

据式(41)有

$$\begin{aligned}
U_A(\bar{R}) &= \frac{t}{\sqrt{n}} \sqrt{\frac{\sum_{i=1}^3 (R_i - \bar{R})^2}{n-1}} \\
&= 2.48 \sqrt{\frac{\sum_{i=1}^3 (R_i - \bar{R})^2}{2}} = 0.483\text{m}
\end{aligned}$$

### (3) $\bar{R}$ 的 B 类不确定度

实验中可取

$$\begin{aligned}
U_B(x_m) &= U_B(x_{m+6}) = U_B(x_{m+12}) \\
&= U_B(x_{m+18}) = 0.004\text{mm}
\end{aligned}$$

$$U_B(R_1) = R_1 U_B(R_1) \quad (44)$$

(3) 总不确定度:

$$U = \sqrt{U_A^2(R) + U_B^2(R)} \quad (45)$$

## 2 实验数据处理

### 2.1 实验数据

表 1 是从第  $m$  环开始,使镜筒沿同一个方向移动,每隔两环测量一次所得的实验数据,实验中的光源为钠光灯,其波长  $\lambda = 5893\text{\AA}$ .

则据式(43)和式(44)得

$$U_B(R_1) = 0.0970\text{m}$$

$$U_B(\bar{R}) = \frac{\sqrt{3}}{3} U_B(R_1) = 0.0560\text{m}$$

### (4) $\bar{R}$ 的总不确定度

将 A 类不确定度和 B 类不确定度代入式(45)得

$$U = \sqrt{U_A^2(R) + U_B^2(R)} = 0.49\text{m}$$

### (5) 测量结果

$$R = \bar{R} \pm U = 2.36 \pm 0.49\text{m}$$

## 参 考 文 献

- [1] 朱鹤年. 物理实验研究. 清华大学出版社, 1994.
- [2] 林 杼, 龚镇雄. 普通物理实验. 人民教育出版社, 1981.
- [3] 潘人培, 董宝昌. 物理实验教学参考书. 高等教育出版社, 1990.